Statistical Significance Based Graph Cut Segmentation for Shrinking Bias

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Abstract. *Graph cut algorithms are very popular in image segmentation approaches. However, the detailed parts of the foreground are not segmented well in graph cut minimization.There are basically two reasons of inadequate segmentations: (i) Data - smoothness relationship of graph energy. (ii) Shrinking bias which is the bias towards shorter paths. This paper improves the foreground segmentation by integrating the statistical significance measure into the graph energy minimization. Significance measure changes the relative importance of graph edge weights for each pixel. Especially at the boundary parts, the data weights take more significance than the smoothness weights. Since the energy minimization approach takes into account the significance measure, the minimization algorithm produces better segmentations at the boundary regions. Experimental results show that the statistical significance measure makes the graph cut algorithm less prone to bias towards shorter paths and better at boundary segmentation.*

Keywords: Graph Cut Segmentation, Energy Minimization, Shrinking Bias, Statistical Significance Analysis

1 Introduction

Current state-of-the-art segmentation methods are based on optimization procedure [1]. One of the popular optimization based methods is graph cut minimization [2]. The graph cut approach models the image segmentation problem as pixel labeling such that each pixel is assigned to a label which denotes the segmentation classes. The algorithm first builds a graph $G = (V, E)$. V consists of set of vertices that correspond to the pixel features (e.g. intensity) and two extra vertices which denote object and background terminals. E consists of edges which are assigned to a nonnegative weights according to the relationship between the vertices. After the graph structure is constituted, the optimal labeling configuration is found by minimizing an energy functional whose terms are based on the edge weights of the graph. The standard graph energy functional is formulated as,

$$E(f) = \sum_{i \in V} E_d(f_i, d_i) + \lambda \sum_{i,j \in N} E_s(f_i, f_j), \quad (1)$$

where V are the vertices, f_i is the segmentation label, d_i is the a priori data of pixel i, and N represents the neighborhood pixels j of pixel i. The first term in the energy functional is called the data term E_d, which confines the segmentation labels to be close to the observed image. The second term is used for the smoothness which confines the neighboring nodes to have similar segmentation labels. The regularization weight λ balances the relationship between the data and smoothness terms.

1.1 Motivation

Graph cut algorithms produce successful solutions for the image segmentation [2–4]. However, the foreground boundary, especially at the detailed parts still cannot be obtained well in the graph cut minimization. There are basically two reasons of inadequate segmentations at the boundary regions:

(i) Data-Smoothness Relationship. One of the reasons of the inadequate segmentation of graph cut algorithms is due to the energy minimization approach. The trade off between the data and the smoothness terms should be well regularized in the energy functional. In order to obtain the boundary of the foreground accurately, regularization should be small. In Fig 1.b, segmentation is obtained with a small λ. Small λ segments the objects sharply, however, it produces noisy solutions (grassy regions). If we increase the λ in order to obtain a noiseless segmentation, this time we lose the details such as the legs and the ears of the horses (Fig 1.c). For the optimal segmentation, λ parameter should be optimal as in Fig 1.d. Even for the optimal segmentation, the detailed parts of the foreground still cannot be segmented accurately. The main reason of the inadequate segmentation in energy minimization approach is that the optimal regularization parameter for overall segmentation is generally high for the boundary regions.

(ii) Shrinking Bias. Another reason of the inadequate segmentation of graph cut minimization is the shrinking bias [5] which is an inherent bias towards shorter paths. The smoothness term in graph-cut methods consists of a cost summation over the boundary of the segmented regions. A short expensive boundary may cost less than a very long cheap one. Especially at the long and thin boundaries of objects, the graph cut algorithms may cut the boundary along the shorter paths which causes inadequate segmentation for those parts. Figure 2 shows the optimal segmentation for the horse image in Figure 1.a and illustrates the shrinking bias problem. The green boundary denotes the ground truth segmentation. However, the graph cut algorithm segments the image along the red boundary. Note the marked regions on the image. The algorithm segments the object at the short-cut boundaries instead of long and thin boundary paths.

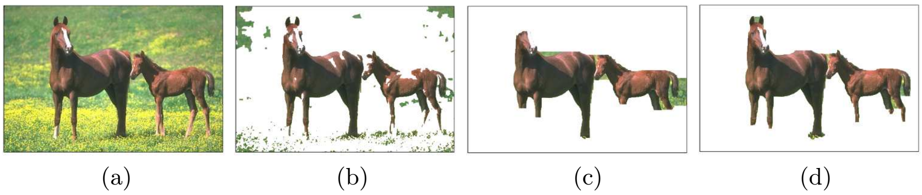


Fig. 1. The illustration of the trade off between the data and the smoothness terms of the graph cut minimization. a) Input image. b) Segmentation by a small λ. Less regularization provides to segment the detailed regions of the foreground such as the legs parts. c) Segmentation by a large λ. Not only the noisy segmentation but also the detailed parts of the segmentation is lost. d) Optimal segmentation is still not well enough at the boundary parts.

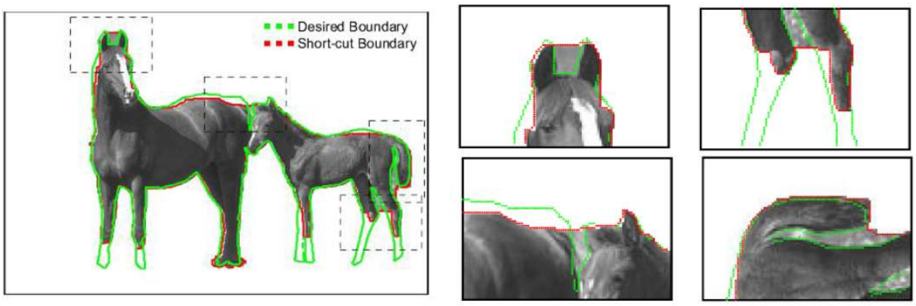


Fig. 2. Graph Cut methods may short-cut the foreground along the red borders instead of following the green borders, because short-expensive boundary may cost less than a very long cheap one.

1.2 Related Work

Shrinking bias problem of graph cuts is first addressed by Kolmogorov and Boykov [5]. They define the flux along the boundary and improve the segmentation. Flux knowledge causes stretching at the boundary while the graph cut algorithm tries to smooth the solution because of the energy minimization. Although the flux integration produces better solutions than the original graph cut approach, the algorithm cannot be extended to color images, because flux can be defined only on the grey-level images [6]. Another work which tries to overcome the inadequate segmentation is geodesic segmentation which avoids the shrinking bias of the graph cut methods by removing the edge component in the energy formulation [7]. However this approach cannot localize the object boundaries and it is very sensitive to seed placement [8]. Vincente and Kolmogorov [6] attempt to solve the long and thin object segmentation by adding connectivity priors. They manually add some additional marks at the endpoints of long-thin objects and then run the Dijkstra's algorithms after the graph cut minimization. Recently, researchers argued that the same λ may not be optimal for all regions of the image. They proposed different algorithms which spatially change the regularization parameter based on the local attributes of the images [9–12]. Since the regularization weight is decreased at the boundary parts, energy minimization cannot over-smooth the thin and long parts of the foreground. Therefore, the spatially-adaptive methods will produce better segmentation results than the traditional graph cut algorithms.

In this work, the statistical significance measure is integrated into the energy minimization approach in order to improve the image segmentation problem. In traditional statistics, statistical significance measures the randomness of an outcome. It is previously proposed that the statistical significance can be used as a comparison measure for the outcomes of different distributions [13, 14]. In this work, we redefine and modify the idea for the shrinking bias problem and include additional experiments. The statistical significance measure is included in the energy minimization approach through the graph structure. We measure the statistical significance of all weights on the graph. Then we reconstruct the graph structure by changing the weights with their statistical significance measurements.

2 Statistically Significant Graph Cut Segmentation

2.1 *p*-value Calculation

Statistical significance is a probability value (*p*-value) which is the measurement of randomness. It is used for the hypothesis testing mechanism in statistics. If the observed outcome of an experiment is statistically significant, this means that it is unlikely to have occurred by chance, according to the significance level which is a predetermined threshold probability.

In order to measure the statistical significance of the outcome of an experiment, cumulative probability distribution function of the experiment should be known. If the distribution of the outcome is a known distribution such as the exponential distribution, the parameters of this distribution is used to measure the significance. On the other hand, if the distribution is not known, the possible outputs of the experiment is used to form the probability distribution. The area under the probability distribution forms the cumulative distribution function. The location of the outcome on cumulative distribution determines the statistical significance of the observed outcome. Equation 2 denotes the statistical significance of the outcome x.

$$F(x) = P(X <= x) = \sum_{-\infty}^{x} P(X = x) \tag{2}$$

$P(X = x)$ is the probability distribution of experiment X, F(x) produces the p-value of the statistic x. If the obtained p-value is small then it can be said that an unusual outcome has been obtained.

2.2 Measuring the significance of edge weights

In this work, we used the significance measure to bring the data and smoothness energy terms (outcomes) into the same base, which is different from the traditional usage. We measured the statistical significance of energy terms in terms of edge weights of the graph structure. In graph cut algorithms, objective function is constituted of the edge weights. The edges between the terminal and pixel vertices are called t-links whose weights form the data energy term. On the other hand, the edges between the neighboring pixel vertices are called n-links whose weights form the smoothness terms of the energy function. The weights of different types of links are determined through different functions such as squared differences, absolute differences, truncated absolute differences, laplacian zero crossing or gradient direction. As an example, in the interactive segmentation of Boykov and Jolly [2], the weights of t-links are based on the marked pixel histogram, whereas, n-links are the intensity difference between the neighboring pixels. Note that, data and smoothness terms of the energy formulation have different functional forms, whereas, graph cut minimization try to minimize the different functional forms simultaneously through the same objective function. In this work, we used the significance measure to bring the energy terms on the common base by expressing the weights in terms of the statistical significance measure.

In order to measure the statistical significance of data and smoothness terms, the probability distribution of the terms should be generated. The edge weights on the graph form the probability distribution of terms. Figure 3 illustrates the procedure. The weights of the t-links (marked as red color on the graph) form the probability distribution of data term of the energy function, on the other hand, the weights of the n-links (marked as blue on the graph) form the probability distribution of smoothness term. Two sample edge weight is denoted on the graph by green color. Then we measure the statistical significance of each edge weight by evaluating the weights on the distributions. t-link weights are evaluated on the data term distribution; n-link weights are evaluated on the smoothness distribution. After measuring each weight significance, we reconstruct a new graph structure in which edge weight is assigned to a significance value.

Equation 3 and Equation 4 formulates the significance measurement.

$$F(x_d) = P(E_d(f,d) <= x_d), \qquad x_d = E(f_i, d_i) \tag{3}$$

$$F(x_s) = P(E_s(f,d) <= x_s), \qquad x_s = E(f_i, f_j) \tag{4}$$

where x_s is the observed data weight, x_s is the observed smoothness weight, $P(E_d(f,d))$ denotes the probability distribution of data weights, and $P(E_s(f,d))$ denotes the probability distribution of smoothness weights.

3 Data-Smoothness Weights Relationship

We measure the statistical significance of each term by evaluating the terms according to the other graph terms. Evaluating the terms on its own distributions

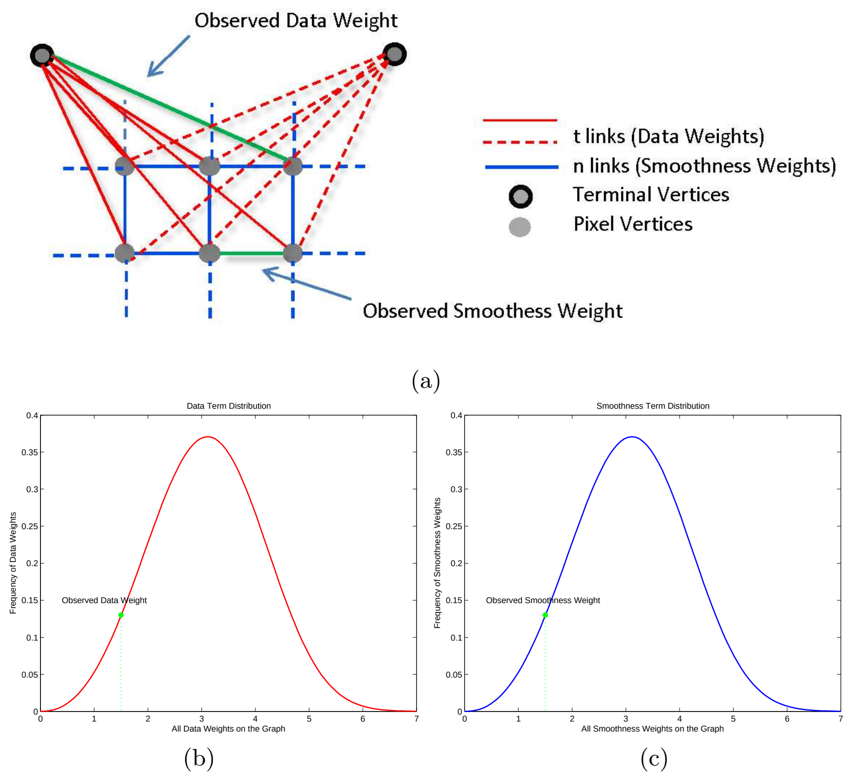


Fig. 3. Data and smoothness weights are normalized by evaluating weights according to other data and smoothness weights on the graph. a) A simple graph structure. Data edges denoted by red color, smoothness edges denoted by blue color. b) Probability distribution of data terms. c) The probability distribution of the smoothness terms.

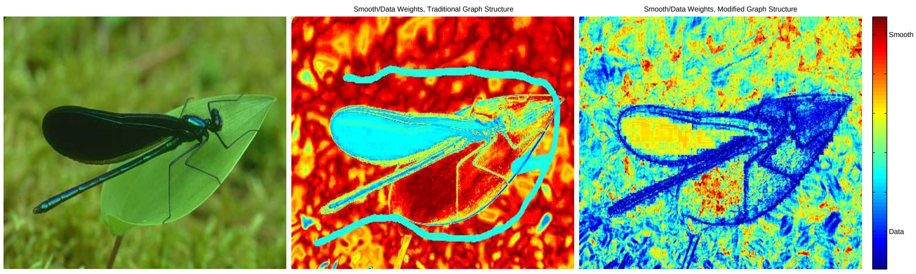
and expressing the edge weights by the same measurement have two explicit advantages:

(i) The significance measure decreases the scale and distribution differences between the data and smoothness energy terms and bring them on similar base. Therefore, the tradeoff between the terms would be properly regularized.

(ii) The significance measure for the data weights are determined according to other data weights on the graph. Similarly, the significance measure for the smoothness weights are determined according to other smoothness weights on the graph. It can be interpreted as each weight is normalized relative to other weights. As an example, if one of the data weight has a high significance among the other data weights, we can say that data term for that pixel is statistically more significant than the smoothness term, albeit both terms have equal weight. Normalization change the relative weights of data and smoothness terms according to their randomness. Rare weights become more important than the normal weights.

In order to show the relative relationship between data and smoothness weights of each pixel, we constructed weight maps for both graph structures. We calculated the relative weight of each pixel $i \in I$ of image I using Formula 5. Then we normalize the weight rates to a fixed range [0-1]. If the weight rate is close to the 1, this means that smoothness weight is relatively bigger than the data weight for that pixel. If the smoothness weight increases, pixel get closer to the red. On the hand, if the weight rate is close to the 0, it can be said that the data weight is more important for that pixels. We show that type of pixels with blue. Figure 4.a denotes the weight map of original graph structure, Figure 4.b denotes the weight map of modified graph structure. Note that data weights at the boundary part takes more importance than the smoothness weights in the modified graph structure.

$$\frac{\lambda E_s(f_i, f_j)}{E_d(f_i, d_i)} \qquad \forall i \in I \tag{5}$$



(a) (b) (c)

Fig. 4. Data-Smoothness relationship for the dragonfly image. (a) A sample image. (b)Smoothness/Data weight rate for each pixel of traditional graph structure. (c) Smoothness/Data weight rate for each pixel of modified graph structure.

4 Improvement in Shrinking Bias Problem

Statistical significance measurement decreases the smoothness weights along the boundary as it can be seen in Figure 4. Therefore finding a short expensive boundary, which may cost less than a very long cheap one become harder. Figure 5 demonstrates the improvement in shortcutting. The segmentation is obtained by minimizing the modified graph cut structure whose weights are calculated by significance measurement. The red contour denotes the short-cut boundary which is the optimal segmentation of traditional graph structure as we showed previously in Figure 2. Note that the blue contour is explicitly closer to the desired boundary.

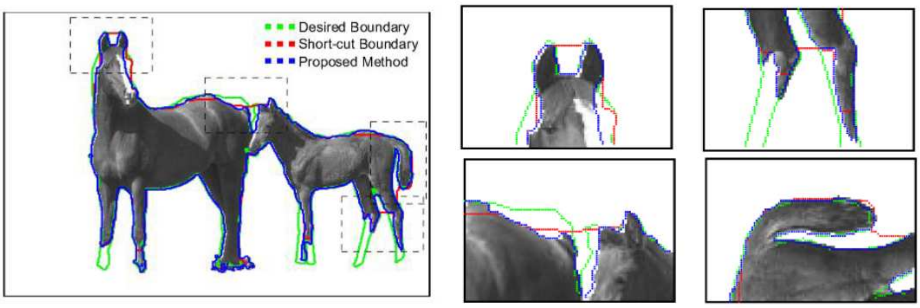


Fig. 5. Improvement in Shrinking Bias problem. The blue contour is closer to the desired boundary than the short-cut boundary.

5 Experimental Results

To quantitatively evaluate the accuracy of the segmentation of the proposed approach, we applied it to the Berkeley dataset [15]. We obtained optimal segmentation of original graph structure and modified graph structure by comparing the segmentations by ground truths. Figure 6 displays some segmentations results of both approaches. Note that the thin and long parts of foreground such as legs of the dragonfly or wood in the bear image. The proposed approach produces better solutions at these problematic parts. The percentage errors of the segmentations are listed on Table 1.

Table 1. Error Rates of the Segmentations in Figure 6

Image	Traditional Graph Structure Optimal Segmentation Error Rate	Modified Graph Structure Optimal Segmentation Error Rate
Dragonfly	1.29 %	1.08 %
Eagle	4.34 %	2.99 %
Horse	2.97 %	2.56 %
Bear	4.32 %	3.07 %
Plane	0.83 %	0.70%
Trees	1.74 %	1.01%

6 Discussion

In this paper we have integrated the statistical significance measure into the graph structure in order to improve the graph cut segmentation approach. We measured the significance of data and smoothness edge weights according to other weights. Then we constructed a new graph structure whose edge weights

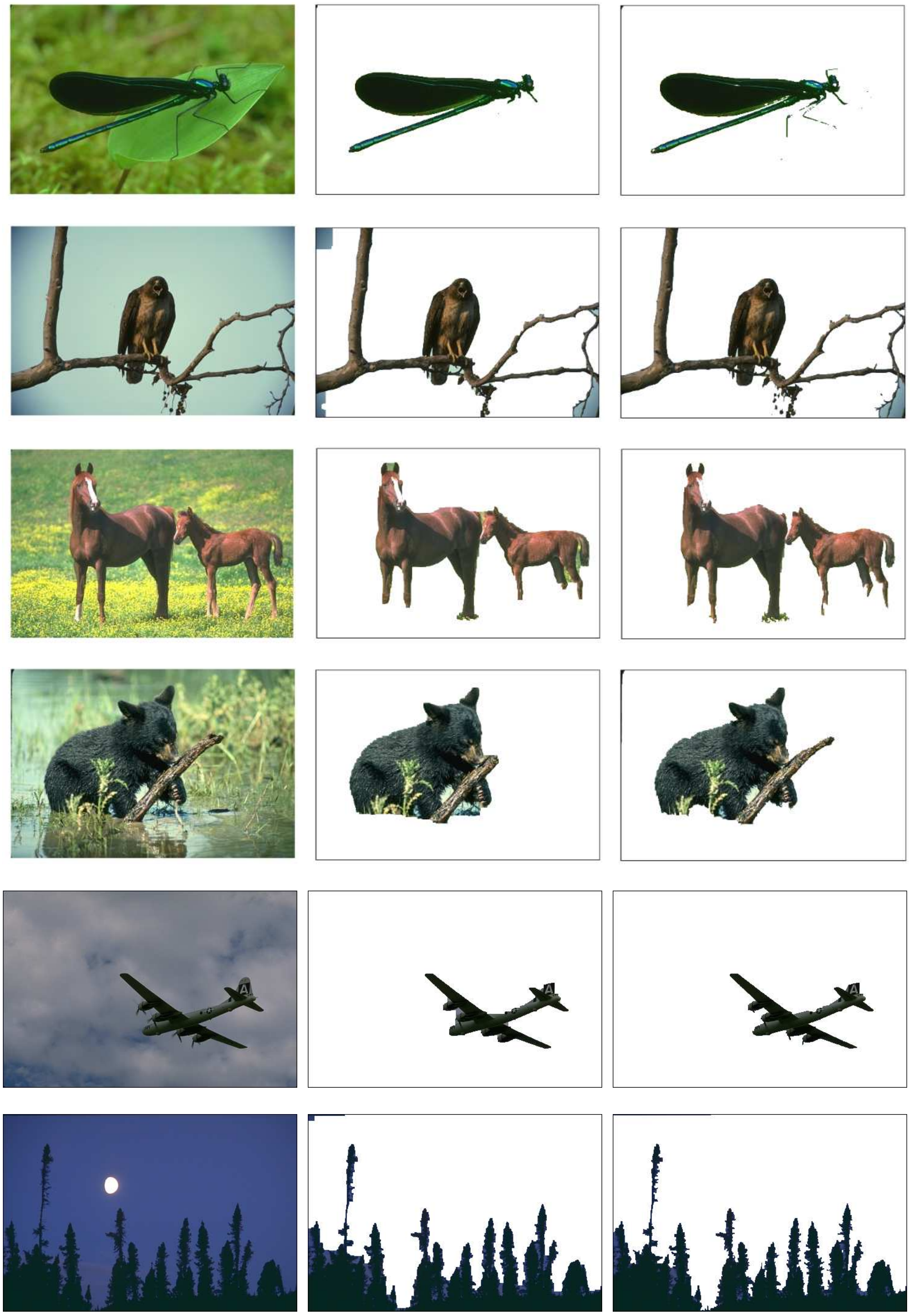


Fig. 6. FirstColumn: Image from the Berkeley set [15]. SecondColumn: Optimal segmentation by traditional graph structure. ThirdColumn: Optimal segmentation by modified graph structure based on statistical significance measurement.

are the significance measurements. Using the significance measurements instead of weights can be interpreted as each weight is normalized relative to other weights. In the new graph structure, the relative weights of data and smoothness edges are changed according to their randomness. Especially at the boundary regions of the foreground, the data weights gets more importance than the smoothness weights. In another word, the smoothness weights along the boundary is decreased. Therefore, finding a short expensive boundary which may cost less than a very long cheap one become harder. We demonstrated our algorithm on several images on Berkeley segmentation set, and showed that our optimal segmentations are better than the optimal segmentations of traditional graph cuts.

References

1. P. Felzenszwalb, R. Zabih, Dynamic Programming and Graph Algorithms in Computer Vision, To appear in the IEEE Transactions on Pattern Analysis and Machine Intelligence
2. Yuri Y. Boykov Marie-Pierre Jolly, Interactive Graph Cuts for Optimal Boundary and Region Segmentation of Objects in N-D Images, ICCV 2001.
3. Y. Boykov, G. Funka-Lea, "Graph cuts and efficient N-D image segmentation", Int. J. Computer Vision. 70: 109-31, 2006.
4. Rother, C., Kolmogorov, V., Blake, A.: Grabcut: Interactive foreground extraction using iterated graph cuts. In: SIGGRAPH, 2004.
5. Kolmogorov, V., Boykov, Y.: What metrics can be approximated by geo-cuts, or global optimization of length/area and flux. In IEEE ICCV, vol.1, pp.564–571, 2005.
6. Vicente, S., Kolmogorov, V., Rother, C.: Graph cut based image segmentation with connectivity priors. In IEEE CVPR, 2008.
7. Bai, X., Sapiro, G.: A geodesic framework for fast interactive image and video segmentation and matting. IEEE ICCV, pp.1–8, 2007.
8. Price, B.L., Morse, B., Cohen, S.: Geodesic Graph Cut for Interactive Image Segmentation. In IEEE CVPR, 2010.
9. J. Rao, G. Hamarneh and R. Abugharbieh, Adaptive Contextual Energy Parameterization for Automated Image Segmentation, In Proc. ISVC, vol.5875, 2009.
10. G. Gilboa, J. Darbon, S. Osher, T. Chan, Nonlocal convex functionals for image regularization, UCLA CAM-report, 2006.
11. S. Candemir, Y. S. Akgul, Adaptive Regularization Parameter for Graph Cut Segmentation, ICIAR. 1: 117-26, 2010.
12. Schoenemann, T., Kahl, F., Cremers, D.: Curvature regularity for region-based image segmentation and inpainting: A linear programming relaxation. In IEEE ICCV, 2009.
13. S.Candemir, Y.S.Akgul. A Nonparametric Statistical Approach for Stereo Correspondence, IEEE conf. Computer and Information Sciences, pp.1-6, 2007.
14. S.Candemir, Y.S. Akgul, Statistical Significance Based Graph Cut Regularization for Medical Image Segmentation, To appear in Turkish Journal of Electrical Engineering and Computer Sciences.
15. The Berkeley Segmentation Dataset and Benchmark Page, http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/