

Adaptive Regularization Parameter for Graph Cut Segmentation

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Abstract. Graph cut minimization formulates the segmentation problem as the linear combination of data and smoothness terms. The smoothness term is included in the energy formulation through a regularization parameter. We propose that the trade-off between the data and the smoothness terms should not be balanced by the same regularization parameter for the whole image. In order to validate the proposed idea, we build a system which adaptively changes the effect of the regularization parameter for the graph cut segmentation. The method calculates the probability of being part of the boundary for each pixel using the Canny edge detector at different hysteresis threshold levels. Then, it adjusts the regularization parameter of the pixel depending on the probability value. The experiments showed that adjusting the effect of the regularization parameter on different image regions produces better segmentation results than using a single best regularization parameter.

Key words: Regularization Parameter, Graph Cut, Image Segmentation.

1 Introduction

The first step of many computer vision systems is the object-background segmentation. For the performance of the advanced steps of the system, the foreground segmentation should be accurate. There are different segmentation techniques in the literature based on clustering [1], curve fitting [2], and energy minimization [3]. The graph cut approach [4, 5] is one of the energy based algorithms which solves the object-background segmentation relatively successfully. The algorithm first builds a graph $G = (V, E)$. V consists of set of nodes that correspond to the image pixels. Two additional nodes are also added to V that represent the object and the background terminals. E are the edges that connect the nodes with nonnegative costs. The optimal segmentation is determined by finding the minimum cost cut on the graph through minimizing the graph energy functional. The standard graph energy functional is formulated as,

$$E(f) = \sum_{i \in V} E_d(f_i, d_i) + \lambda \sum_{i, j \in N} E_s(f_i, f_j), \quad (1)$$

where V is the nodes on the graph, f_i is the segmentation label and d_i is the a priori data of pixel i , and N represents the neighborhood pixels j of pixel i . The first term in the energy functional is called the data term E_d , which confines the segmentation labels to be close to the observed image. The second term is used for the smoothness which confines the neighboring nodes to have similar segmentation labels.

The data term of the energy formulation is inadequate to obtain a successful segmentation because of the ill-posed nature [6, 7] of the segmentation problem. Therefore, energy minimization approaches constrain the solution space by adding a smoothness term. The smoothness term is included in the energy formulation through a regularization parameter λ which determines the degree of the smoothness of the solution. Choosing a suitable λ is important to obtain a meaningful solution. If λ is small, the segmentation will be noisy; on the other hand, if λ is large, the segmentation will not fit the observed data. Figure 1 illustrates the trade-off between the data and smoothness terms on a graph cut minimization. The segmentation with a small regularization parameter (Fig 1.b) produces noisy solutions (grassy regions). If we increase the regularization parameter in order to obtain a noiseless segmentation, this time we lose the details such as the legs and the ears of the horses (Fig 1.c). The better segmentation is obtained with the most suitable regularization parameter (Fig 1.d). However, it still has problems on some parts of the foreground. Note that the ear and the tail regions of the horses are over-segmented (red marked regions); the legs of the horses are under-segmented (blue marked regions).

This paper introduces a new idea that, the trade-off between the data and the smoothness terms should not be balanced by the same regularization parameter for the whole image. For example, the grassy part of the image in Figure 1.a needs higher regularization than the leg part of the horses. However, using the same λ makes the smoothness effect equal on the whole image, and causes over/under-segmented regions even with the most suitable regularization parameter. We propose a method that adaptively changes the regularization parameter of the graph cut minimization depending on the image regions. The method determines the foreground boundary of the image. Then, it adjusts the regularization parameter for each pixel based on the probability of that pixel being part of the boundary.

We introduce the method on interactive graph cut [8, 9] which is one of the convenient and widely used graph cut approaches. The user initially marks some pixels as object and some pixels as background to direct the graph cut algorithm. The marked pixels form the object and background intensity distributions. Then, the data and the smoothness energy terms are formulated using these distributions. In order to validate the proposed idea, we implemented a series of experiments on the interactive graph cut minimization by applying our method. The experiments showed that, adjusting the effect of the regularization parameter on different regions of the image produced better segmentation results than using a single best regularization parameter.

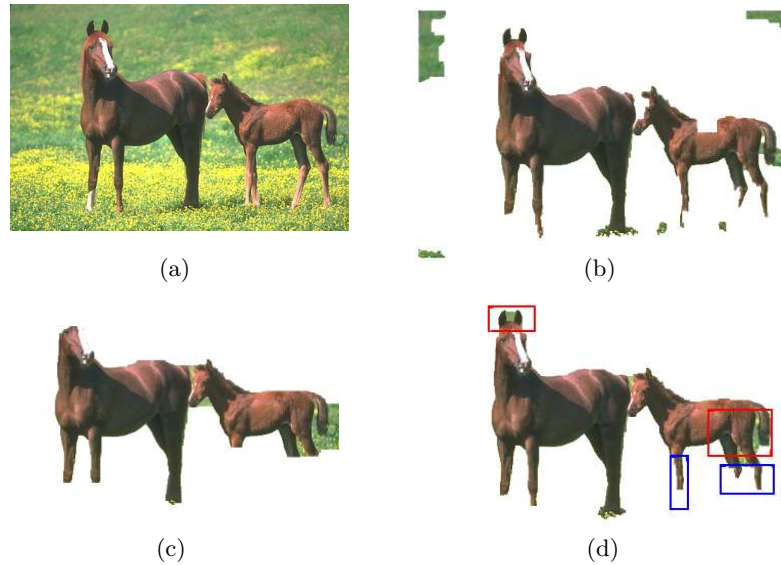


Fig. 1. The illustration of the trade of between the data and the smoothness terms of the graph cut minimization. a) Input image. b) Segmentation is obtained with a small λ , where the smoothness term has small importance in graph energy formulation. c) Segmentation is obtained with a large λ , where the smoothness term has significant importance in the graph energy formulation. d) The best segmentation with the most suitable regularization parameter. Red marked area denote the over segmentations; blue marked area denote the under segmentations.

The rest of the paper is organized as follows: The next section summarizes the related work on the regularization parameter. We introduce the method in Section 3. We show the experimental results in Section 4. Section 5 provides discussions and concluding remarks.

2 Related Work

Obtaining a suitable regularization parameter is as old as the regularization problem [10]. Researchers proposed general techniques such as L-curve method, discrepancy principle, cross-validation principle [11], and U-curve [12]. Besides the general methods, problem-specific techniques are proposed such as for snakes [13] and for image restoration [14]. However, it is recently realized that the suitable regularization parameter depends on statistics of image noise and variation of scene structures [15, 16]. Therefore, different image sets need different regularization parameters for an optimal performance. A few papers focused on the optimal choice of regularization parameter from the observed image. Zhang and Seitz [16] proposed a probabilistic mixture model for the λ determination of a stereo pair. Peng and Veksler [17] proposed a regulariza-

tion parameter selection method for the segmentation problem. Although these approaches estimate suitable parameters, they produce a single regularization parameter for the whole image. Our method on the other hand, claims that the regularization parameter between the energy terms should not be the same for all regions of the image. Different parts of the image needs different regularization effects for a better performance. To the best of our knowledge, our system is the first one to attempt to adaptively regularize the trade-off between the data and the smoothness terms in graph cut segmentation.

As mentioned in the introduction section, using the same λ for the whole image sometimes over-segment the foreground, at other times under-segment such that small foreground details are lost. Vincente and Kolmogorov [18] realized that the graph cut algorithm produces inadequate segmentation for the thin and elongated objects. They proposed a method in which the user marks some additional connectivity priors. Then the algorithm runs the Dijkstra’s algorithm for the thin parts of the objects separately. Their work segments the thin parts of the objects successfully. However, it needs additional marking and increases the computational load because of the run-time costs of the Dijkstra’s algorithm. Moreover, their approach did not solve the over-segmented part of the segmentation such as the tail part of the horses.

3 Proposed Method

Graph cut approach solves the segmentation problem by minimizing an energy functional which is the linear sum of the problem constraints. The data constraint is inadequate to obtain a unique solution, therefore, the smoothness constraint is included to the energy formulation through a regularization parameter. The general approach is using a single suitable λ for the graph energy formulation, however, we propose that the effect of the regularization should not be the same for the whole image.

We propose a method which arranges the effect of the regularization parameter on different parts of the image. The method first calculates the edge pixels of the image by the Canny edge detector. We run the edge algorithm on the observed image at different hysteresis threshold levels between [0.1-1] (the maximum is 1). Then we calculate the edge probability of each pixel by the linear average of edge maps such that $\bar{I} = \frac{1}{n} \sum_{k=1}^n I_k$, where I_k is the binary edge map at hysteresis threshold set k . If pixel i is labeled as an edge pixel for the most of the threshold levels, it has a high probability of being an edge pixel. We illustrated the probability calculation of each pixel in Figure 2. Figure 2.b shows the probabilities of being an edge pixel. In order to decrease the smoothness effect at the near boundary regions, we convolve the probability map with a gaussian kernel (Figure 2.c).

The proposed algorithm modifies the regularization parameter by reducing the smoothness terms of the pixels on the boundary regions. We formulated the

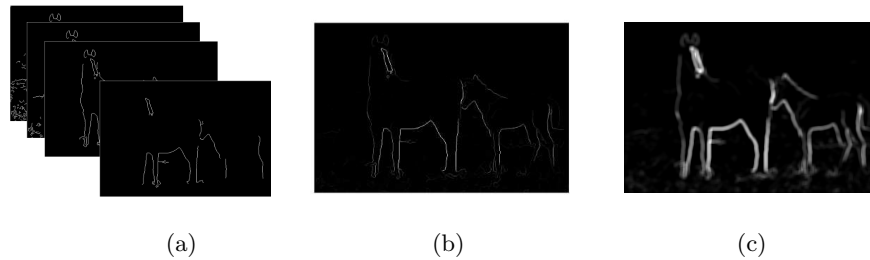


Fig. 2. a) The edge maps of horse image in fig.1 at different hysteresis threshold levels of Canny edge detector. b) The edge probability map of the horse image is obtained by the linear average of the edge maps at threshold levels between [0.1-1]. c) The probability map is obtained by gaussian smoothing in order to decrease the smoothness effect on the near-boundary regions.

proposed idea as

$$E(f) = \sum_{i \in V} E_d(f_i, d_i) + (1 - \bar{I}_i) \lambda \sum_{i, j \in N} E_s(f_i, f_j), \quad (2)$$

where \bar{I}_i is the edge probability of pixel i . If the probability of being an edge of the pixel i is high, the regularization parameter will be multiplied with a small value. Therefore, the smoothness effect will decrease for the pixel i . Similarly, if the probability of being an edge is small, we multiplied the smoothness term with a larger value.

4 Experiments

In order to validate the proposed idea, we implemented the interactive graph cut algorithm and segmented the images in Berkeley data set [19]. We first marked some pixels as object and some pixels as background on the image. We constructed the graph structure using the marked pixel histograms. Then we minimized the graph using the energy formulation of Eq. 1 with the regularization parameters between 0 and 99. We calculated the percentage errors of segmentations for each regularization parameter by comparing the obtained labeling with the ground truth segmentation.

We segmented the same images using the proposed approach. We first obtained edge maps of the images using the Canny edge detector at different hysteresis threshold values. Then, we calculated the edge probability of each pixel of the observed image. We minimized the graph structure using the energy formulation of Eq. 2. For a reliable comparison, we used the same graph structure for both approaches. Figure 3 shows the segmentations of both approaches for the best λ values. Note that the tail and the ear regions of the horses are not segmented accurately even using the best λ , which was chosen manually. Some of the leg pixels are also labeled as background (Fig. 3b). The similar corruptions are observed for the other images. The best λ is inadequate to properly

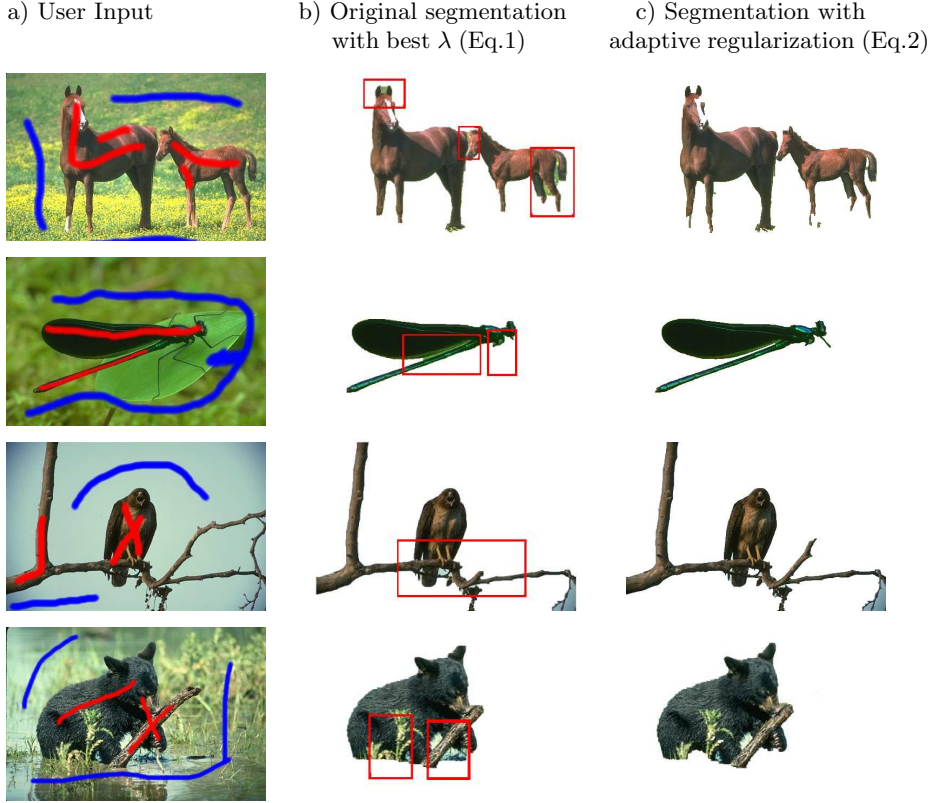


Fig. 3. Comparison of the original graph cut segmentation with the proposed approach. a) User Input. Red marks denote object pixels, blue marks denotes background pixels. b) Original Segmentation with the best λ . Over and under segmented regions are denoted in rectangles. c) Segmentation with adaptive regularization.

segment the region between the wing and the tail of the insect. Some of the background regions, especially at the near boundary parts of the eagle and the bear are segmented as foreground. The proposed approach on the other hand, adaptively adjusts the regularization parameter based on the edge probability. Since the effect of the regularization parameter is decreased on the edge parts of the image, the segmentation solution is not over-smoothed on the thin and elongated parts of the foreground (Fig. 3c).

We illustrate the reversed normalized edge maps in Figure 4 which represent the smoothness proportion of image regions. The dark intensities denote the image parts which should have smaller regularization parameter. The lighter intensities on the other hand should be more regularized in the energy formulation.

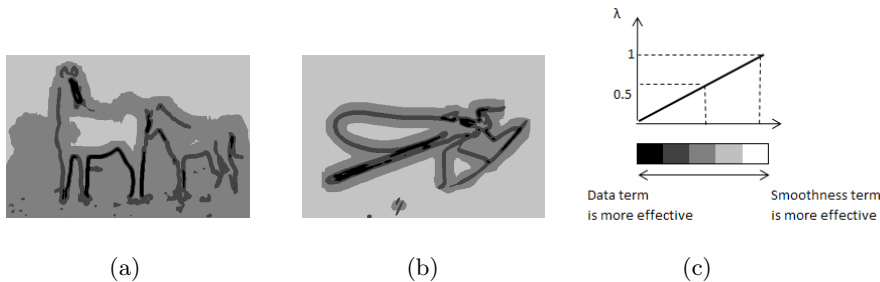


Fig. 4. Illustration of the adaptive regularization values. (a) and (b) are the reversed normalized edge maps of the images in Figure 3. (c) represents the effectiveness of the regularization parameter on the different image regions.

We formulated the proposed method based on the boundary knowledge of the observed image. However, the boundary regions cannot be accurately determined. For example, the back parts of the horses in Figure 4.a cannot be determined. In some part of the images, on the other hand, the unnecessary edges mislead the proposed algorithm (the edge leaf in the insect image Figure 4.b). As a result, the proposed method is influenced by the boundary accuracy. In order to observe the effect of the proposed idea with a better boundary knowledge, we used the ground truth edges of the images. We decrease the effect of the regularization parameter for the boundary and the near-boundary regions of the images manually. Then we used these regularization parameters for the graph cut minimization. The regularization parameters based on the edge maps in Figure 5.a increase the quality of the segmentations (the legs of the horse and the legs of the insect)(Figure 5.b).

We also compared the percentage errors of both approaches using the ground truth segmentation(Figure 6). The red curve denotes the percentage errors of segmentations which are obtained by graph cut minimization in Eq.1. The minimum point of this curve is the best λ for the observed image. The green curve on the other hand, denotes the percentage errors of segmentations which are obtained by the proposed approach (Eq.2). The graphs show that the proposed method performs better than the original graph cut segmentation for the meaningful ranges of the regularization parameter.

5 Discussion

For the graph cut based segmentation tasks, the trade-off between the data and the smoothness terms should not be balanced by the same λ value for the whole image. The boundary regions should be less regularized than the other regions of the image. We proposed a method which adaptively changes the regularization parameter. The method first determines the boundary regions through the Canny edge detector at different hysteresis threshold levels. The linear average of the edge maps produces the edge probability of each pixel. The

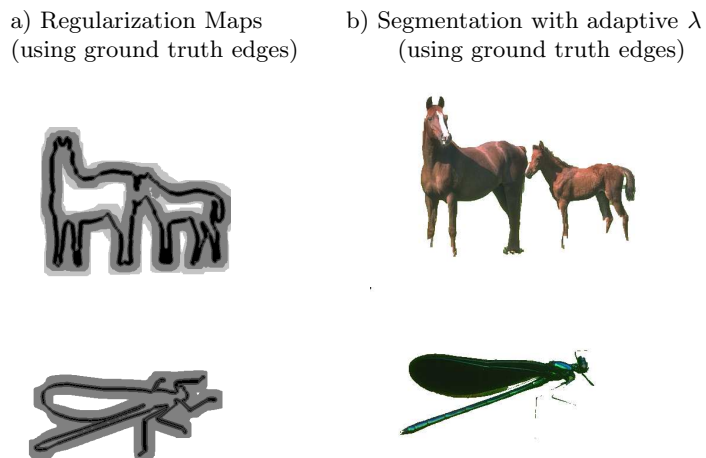


Fig. 5. Segmentation results with the ground truth boundary information.

proposed method adjusts the effect of the regularization parameter using the probabilities of pixels being part of the boundary. Experimental results showed that the proposed method produces better segmentation results than the original graph cut approach for the best λ .

One of the novelties of this paper is the idea that using adaptive regularization parameters for the different parts of the image improves the segmentation result than using a single regularization parameter. The proposed method is the another novelty of the paper. To the best of our knowledge, our system is the first one to attempt to adaptively regularize the trade-off between the data and the smoothness terms in graph cut segmentation.

The method adjusts the regularization effectiveness depending on the probability of pixels being on the boundary region. If the boundary regions are determined incorrectly, the smoothness effect would not be adjusted properly. In the experiments, we used only the edge knowledge for the boundary probabilities. However, the intensity variations cannot not yield reliable boundary probabilities at all times. Therefore, the proposed method should be improved using the other features of the image for the boundary probability calculation, such as the texture and color features of the image.

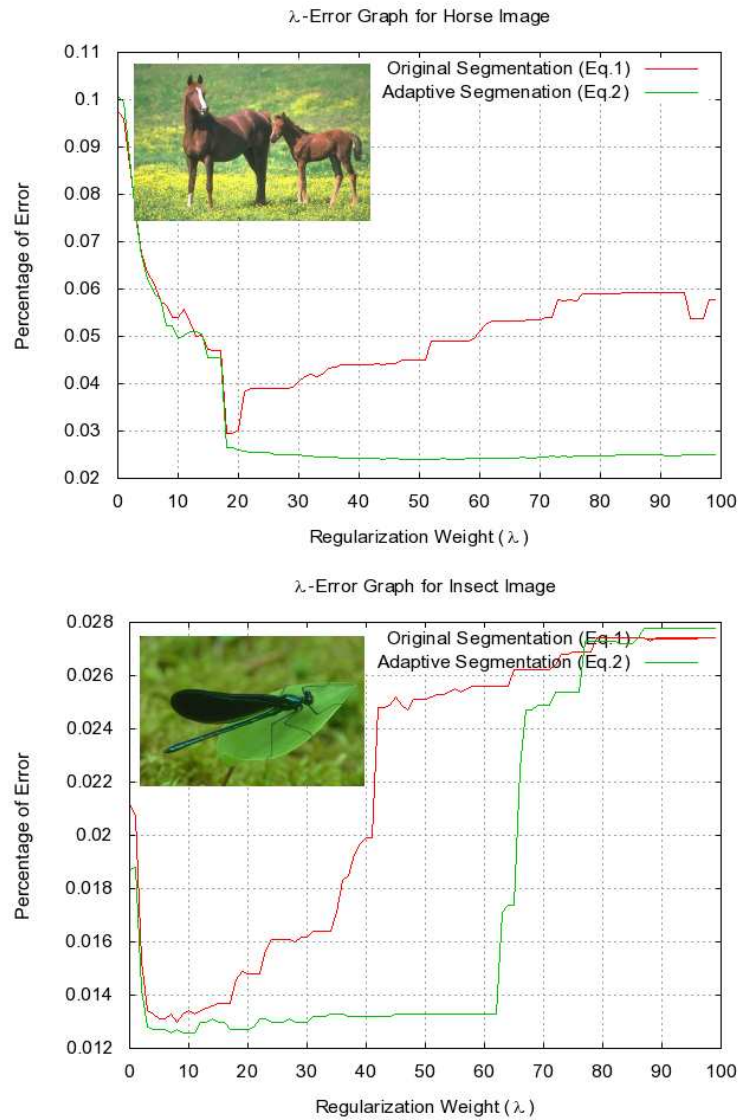


Fig. 6. Comparison of the original graph cut segmentation with the proposed approach for the whole ranges of regularization parameter between 0 and 99.

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